Physics and geometry of knots-quivers correspondence

Piotr Kucharski

Uppsala University, Sweden

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Collaboration

This talk is based on [1810.xxxxx] with:

Pietro Longhi

Tobias Ekholm

with crucial references to [1707.02991, 1707.0417] with:

Markus Reineke

Marko Stošić

Piotr Sułkowski

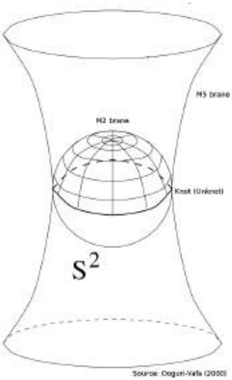
Outline

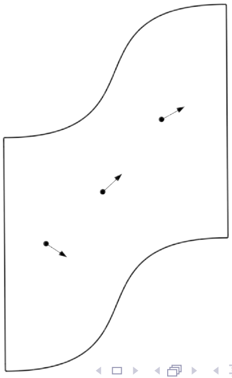
Outline

Setting the stage

- Ooguri-Vafa M-theory system:

Spacetime	Resolved conifold	x	$\mathbb{R}^{1,4}$
M5 brane	Lagrangian submanifold L_K	x	$\mathbb{R}^{1,2}$





Calabi-Yau side (resolved conifold)

- Topological strings $\leftrightarrow$ Chern-Simons theory $\leftrightarrow$ $\leftrightarrow$ Knot theory [Witten]
- Partition function=HOMFLY-PT gen. series

$$Z = P^K(x,a,q) = \sum_{r=0}^{\infty} P_r^K(a,q)x^r$$

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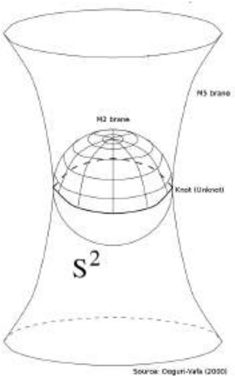
M5 brane	Lagrangian submanifold L_K
M2 branes	Holomorphic curves



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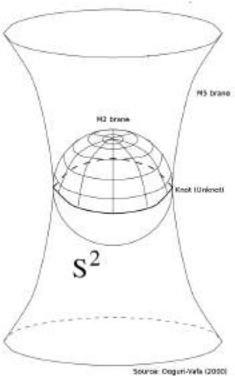

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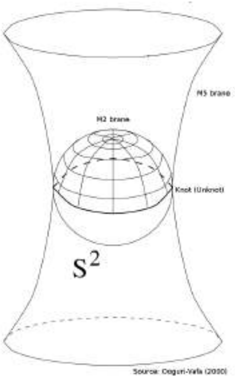


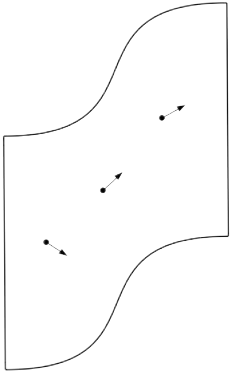
Flat side ($\mathbb{R}^{1,4}$)

- $T[L_K]$: 3d $\mathcal{N} = 2$ eff. theory on $\mathbb{R}^{1,2}$
- Partition function=generating function of Labastida-Mariño-Ooguri-Vafa invariants $N^K_{r,i,j}$

$$Z = \mathrm{Exp}\left(\frac{\sum_{r,i,j} N^K_{r,i,j} x^r a^i q^j}{1-q^2}\right)$$

M5 brane	$\mathbb{R}^{1,2}$
M2 branes	BPS states

- BPS states are counted by LMOV invariants $\Rightarrow$ LMOV integrality conjecture



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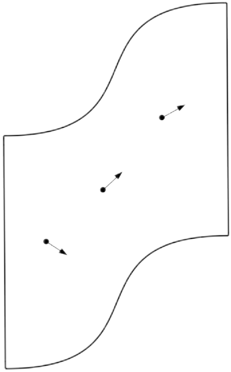

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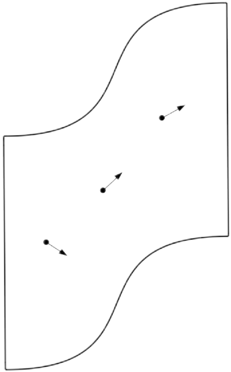

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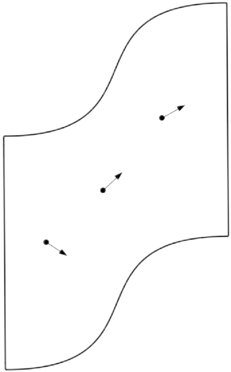

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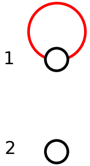
Outline

Quivers

- Quiver $Q = (Q_0, Q_1)$
  - Q_0 is a set of vertices
  - Q_1 is a set of arrows between them (loops allowed)
- Matrix form: C_{ij} gives the number of arrows between vertices i and j
  - If $C_{ij} = C_{ji}$ we call the quiver symmetric
- Quiver representations: $(Q_0, Q_1) \longrightarrow$ (Vector spaces, Linear maps)
- Example from the picture:
  - $Q_0 = \{1, 2\} \longrightarrow$ vector spaces V_1 and V_2 of dimensions d_1 and d_2 (dimension vector $\boldsymbol{d} = (d_1, d_2)$)
  - $Q_1 = \{1 \to 1\} \longrightarrow$ linear map $V_1 \to V_1$



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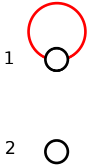

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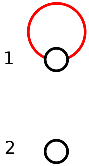

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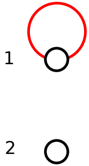

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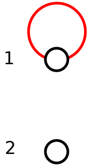

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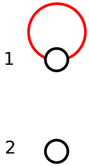
Motivic generating series and DT invariants

- Motivic generating series encodes information about moduli spaces of representations of Q

$$P^{Q}(\mathbf{x},q) = \sum_{d_1,\dots,d_{|Q_0|}\geq 0} (-q)^{\sum_{1\leq i,j\leq |Q_0|} C_{i,j} d_i d_j} \prod_{i=1}^{|Q_0|} \frac{x_i^{d_i}}{(q^2;q^2)_{d_i}}$$

$$= \mathrm{Exp}\left(\frac{\Omega^{Q}(\mathbf{x},q)}{1-q^2}\right)$$



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  - $\Omega^{Q}(\mathbf{x},q)$ is a generating function of motivic Donaldson-Thomas (DT) invariants
  - Exp is the plethystic exponential

$$\mathrm{Exp}\left(x^r a^i q^j\right) = (1 - x^r a^i q^j)^{-1}$$
$$\mathrm{Exp}(f+g) = \mathrm{Exp}(f)\,\mathrm{Exp}(g)$$

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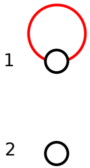

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  - $\Omega^Q(\mathbf{x},q)$ is a generating function of motivic Donaldson-Thomas (DT) invariants
  - $(q^2;q^2)_{d_i}$ is the q-Pochhammer symbol

$$(z;q^2)_r := \prod_{i=0}^{r-1}(1-zq^{2i}) = (1-z)\left(1-zq^2\right)\ldots\left(1-zq^{2(r-1)}\right)$$

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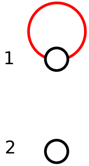

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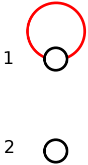
Outline

Knots-quivers correspondence



- Knots-quivers (KQ) correspondence is an equality

$$P^K(x,a,q) = P^{Q_K}(\mathbf{x},q)\Big|_{x_i=a^{a_i}q^{q_i-C_{ii}}x}$$

- $P^K(x,a,q)$ – HOMFLY-PT generating series of knot K
- $P^{Q_K}(\mathbf{x},q)$ – motivic generating series of respective quiver Q_K
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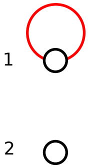

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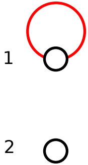

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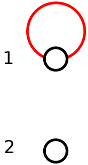

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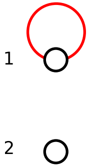
KQ correspondence - unknot example

$$P^{0_1}(x,a,q) = \sum_{r=0}^{\infty} \frac{(a^2;q^2)_r}{(q^2;q^2)_r} x^r$$
$$= 1 + \frac{1-a^2}{1-q^2} x + \dots$$



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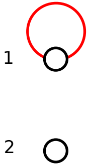

- We can check that

$$P^{0_1}(x,a,q) = P^{Q_{0_1}}(x_1,x_2,q)\Big|_{x_1=a^2q^{-1}x,\ x_2=x}$$

$$1+\frac{1-a^2}{1-q^2}x+\ldots = 1+\frac{-qx_1+x_2}{1-q^2}+\ldots\Bigg|_{x_1=a^2q^{-1}x,\ x_2=x}$$

- Comparing with $x_i = a^{a_i}q^{q_i-C_{ii}}x$ we get

$$(a_1,a_2)=(2,0) \qquad (q_1,q_2)=(0,0)$$

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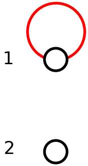

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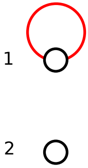


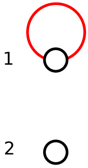
- We can also look at the level of BPS states:

$$\mathrm{Exp}\left(\frac{N^K(x,a,q)}{1-q^2}\right) = P^K = P^{Q_K} = \mathrm{Exp}\left(\frac{\Omega^{Q_K}(\mathbf{x},q)}{1-q^2}\right)\Bigg|_{x_i = a^{a_i} q^{q_i - C_{ii}} x}$$

$$N^K(x,a,q) = \Omega^{Q_K}(\mathbf{x},q)\Big|_{x_i = a^{a_i} q^{q_i - C_{ii}} x}$$

- Integrality of DT invariants for symmetric quivers [Kontsevich-Soibelman, Efimov] implies LMOV conjecture

KQ correspondence and BPS states



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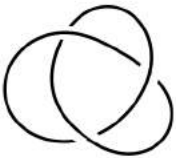
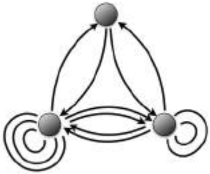
Outline

KQ correspondence discussed so far





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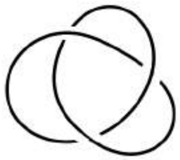
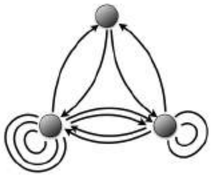
Math	HOMFLY-PT gen. series	$\rightarrow$	Motivic gen. series
Physics			

Towards physics of KQ correspondence





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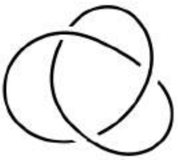
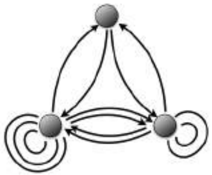
Physics			

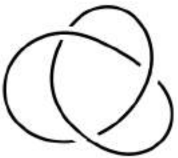
Towards physics of KQ correspondence

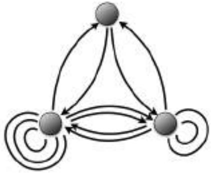




	Knots		Quivers
Math	HOMFLY-PT gen. series	$\rightarrow$	Motivic gen. series
	$\downarrow$		
Physics	3d $\mathcal{N} = 2$ $T[L_K]$		

Missing element





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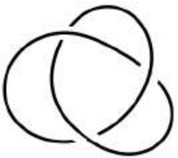

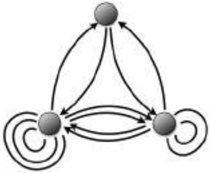

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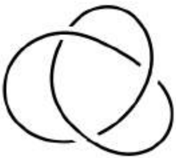

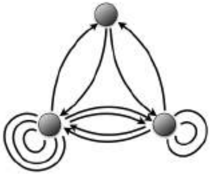

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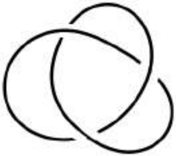
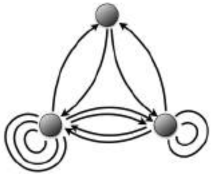
Geometric interpretations





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	Geometric interpretations	
	$\swarrow$	$\searrow$
Physics	3d $\mathcal{N} = 2$ $T[L_K]$	3d $\mathcal{N} = 2$ $T[Q_K]$

Outline

Construction of knot complement theory

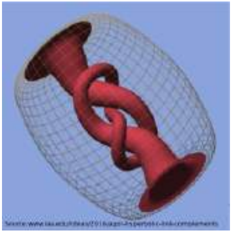
- We can construct 3d $\mathscr{N}=2$ knot complement theory $T[M_K]$ basing on large colour and classical limit of HOMFLY-PT polynomial [Fuji, Gukov, Sułkowski]

$$P_r^K(a,q) \xrightarrow[q^{2r}\ \text{fixed}]{\hbar\to 0} \exp\left[\frac{1}{2\hbar}\left(\widetilde{\mathscr{W}}_{T[M_K]}+O(\hbar)\right)\right]$$

- $\widetilde{\mathscr{W}}_{T[M_K]}$ is a twisted superpotential

$$\mathrm{Li}_2(\ldots) \longleftrightarrow \text{chiral field}$$

$$\frac{\kappa}{2}\log(\ldots)\log(\ldots) \longleftrightarrow \text{CS coupling}$$

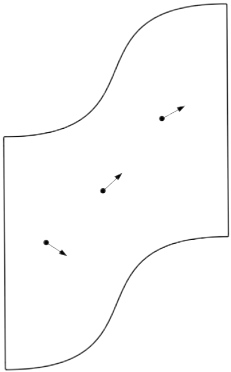


Recall: what is $T[L_K]$

- $T[L_K]$: 3d $\mathcal{N} = 2$ eff. theory on $\mathbb{R}^{1,2}$
- Partition function=generating function of Labastida-Marino-Ooguri-Vafa invariants $N^K_{r,i,j}$

$$Z = \mathrm{Exp}\left(\frac{\sum_{r,i,j} N^K_{r,i,j} x^r a^i q^j}{1-q^2}\right)$$

M5 brane	$\mathbb{R}^{1,2}$
M2 branes	BPS states
- BPS states are counted by LMOV invariants $\Rightarrow$ LMOV integrality conjecture



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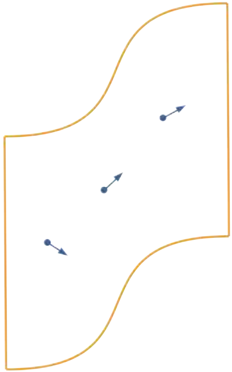


Construction of $T[L_K]$ theory

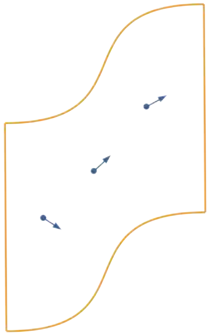
Idea

We construct $T[L_K]$ analogously to $T[M_K]$, but using $P^K(x,a,q)$

$$P^K(x,a,q) \xrightarrow[q^{2r}\to y]{\hbar\to 0} \int dy \exp\left[\frac{1}{2\hbar}\left(\widetilde{\mathscr{W}}_{T[L_K]} + O(\hbar)\right)\right]$$

$$\widetilde{\mathscr{W}}_{T[L_K]} = \widetilde{\mathscr{W}}_{T[M_K]} + \log x \log y$$

- Integral $\int dy$ means that $U(1)$ symmetry corresponding to fugacity y is gauged
- We have the same matter content with just extra CS coupling with background field



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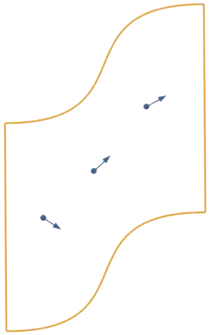

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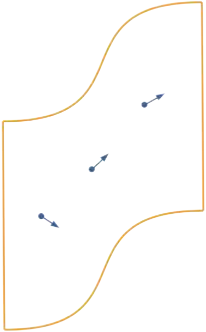

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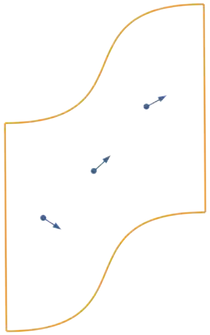

Example: $T[L_{0_1}]$

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- $T[L_{0_1}]$ is a $U(1)$ gauge theory (fugacity y) with one fundamental and one antifundamental chiral
- Antifundamental chiral is charged under the $U(1)_a$ global symmetry arising from S^2 in the conifold

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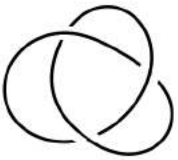
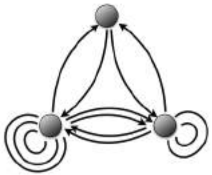
Recall of general idea - now we are here





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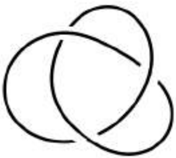

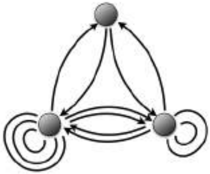

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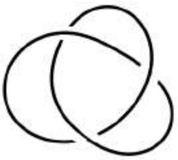

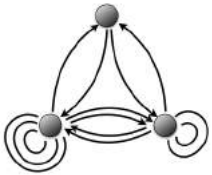

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Missing element: $T[Q_K]$ theory

Idea

Consider large colour and classical limit of motivic generating series!

$$P^{Q_K}(\mathbf{x},q) \xrightarrow[q^{2d_i}\to y_i]{\hbar\to 0} \int \prod_i dy_i \exp\left[\frac{1}{2\hbar}\left(\widetilde{\mathscr{W}}_{T[Q_K]} + O(\hbar)\right)\right]$$

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- Gauge group: $U(1)^{\#\mathrm{vertices}}$
- Matter content: one chiral for each vertex
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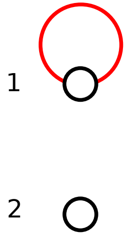

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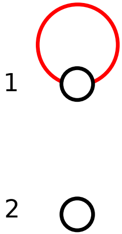

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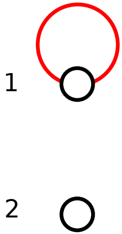

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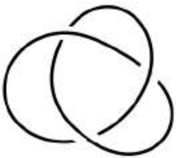

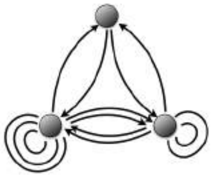

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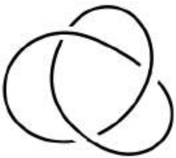
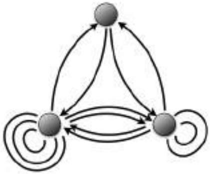
We want the physical interpretation of KQ correspondence





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Duality

- Physical meaning of the knots-quivers correspondence is a duality between two 3d $\mathcal{N} = 2$ theories:

$$T[L_K] \longleftrightarrow T[Q_K]$$

- We already know that

$$Z(T[L_K]) = P^K(x, a, q)$$

- We can calculate the partition function of $T[Q_K]$ and indeed

$$Z(T[Q_K]) = P^{Q_K}(\mathbf{x}, q)$$

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Outline

Quiver nodes extracted from motivic generating series

- Let's look at motivic generating series restricted to $|\mathbf{d}| = 1$

$$P^{Q_K}_{|\mathbf{d}|=1}(\mathbf{x}, q) = \sum_{i \in Q_0} \frac{(-q)^{C_{ii}} x_i}{1 - q^2}$$

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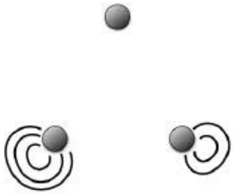

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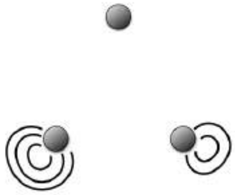

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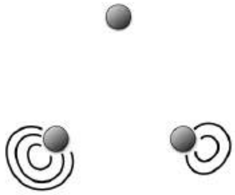

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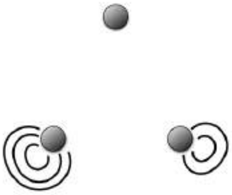

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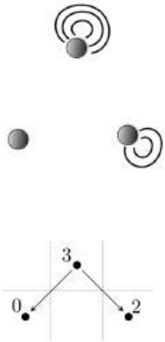
Quiver nodes as homology generators



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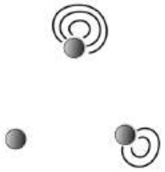

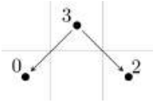

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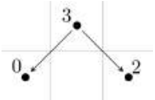

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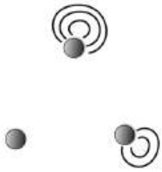

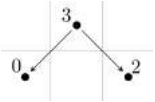

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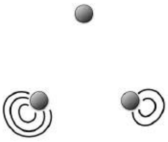
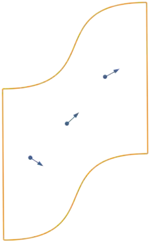
Quiver nodes as BPS states





- Generators of uncoloured homology correspond to BPS states [Gukov, Schwarz, Vafa]
- Quiver nodes correspond to BPS states of $T[L_K]$ counted by LMOV invariants

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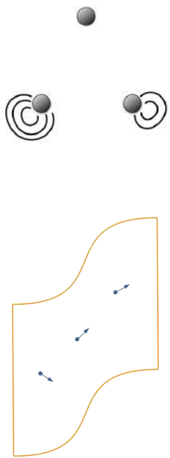

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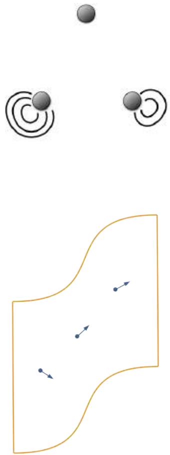

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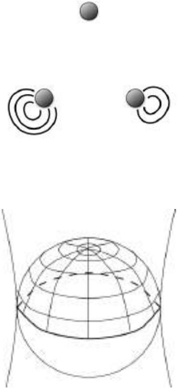

- Recall: M2 branes are BPS states on the flat side and holomorphic disks on the Calabi-Yau side
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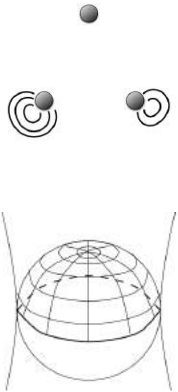

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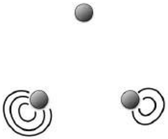

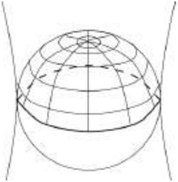

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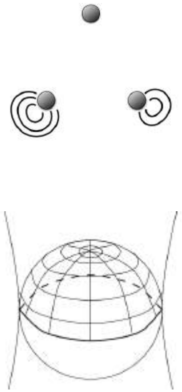

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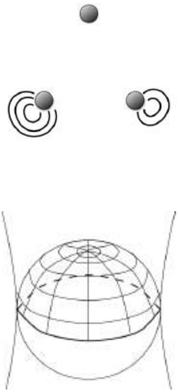

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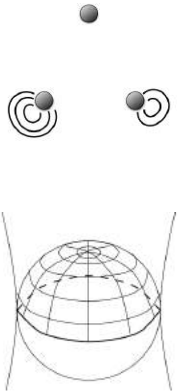

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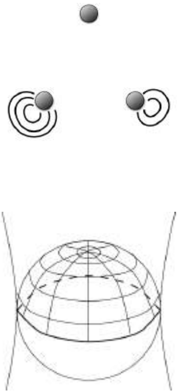

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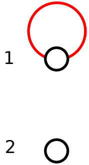

- $\mathscr{H}(0_1)$ has two generators with degrees

$$(a_1,a_2)=(2,0),\quad (q_1,q_2)=(0,0)$$
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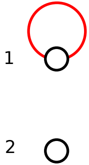

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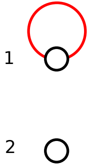

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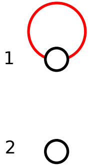

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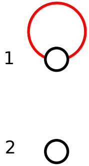

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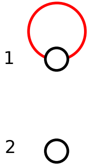

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- Two BPS states on the flat side $\longleftrightarrow$ two holomorphic disks on the Calabi-Yau side
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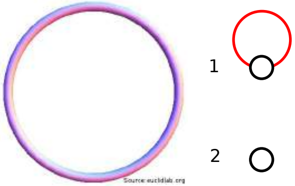

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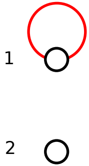
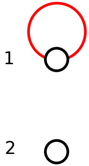


Example: unknot quiver vertices and holomorphic disks

- Two BPS states on the flat side $\longleftrightarrow$ two holomorphic disks on the Calabi-Yau side
- For the second we have the following intepretation of topological data in KQ change of variables:

- $r = 1 \longrightarrow$ winding around L_K
- $a_2 = 0 \longrightarrow$ wrappings around base S^2
- $q_2 = 0 \longrightarrow$ invariant self-linking
- $C_{22} = 0 \longrightarrow$ linking between disk and its small shift

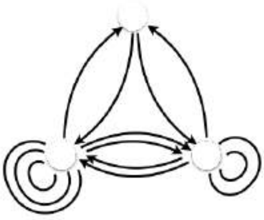


Quiver arrows extracted from motivic generating series

- Let's look closer at motivic generating series

$$P^Q(\mathbf{x},q) := \sum_{d_1,\ldots,d_{|Q_0|}\geq 0} (-q)^{\sum_{1\leq i,j\leq |Q_0|} C_{i,j} d_i d_j} \prod_{i=1}^{|Q_0|} \frac{x_i^{d_i}}{(q^2;q^2)_{d_i}}$$

- Dimension d_i encodes the number of factors corresponding to i-th vertex
- #arrows $C_{i,j}$ encodes interactions between vertices i & j
- Motivic generating series counts all objects that can be made from basic ones (nodes) according to quiver arrows



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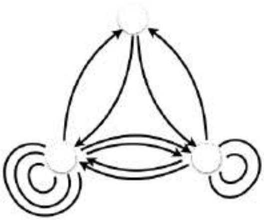

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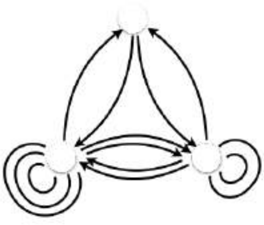

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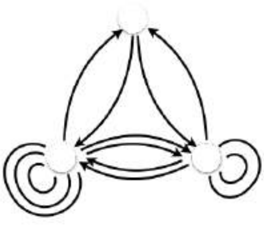

Quiver arrows as disk intersections

- Geometrically $P^Q(\mathbf{x}, q)$ counts all holomorphic curves that can be made from basic disks according to quiver arrows
- Dimension vector $d_i \longrightarrow$ #copies of the disk
- #arrows $C_{i,j} \longrightarrow$ disk boundaries linking#
- Example: one pair of arrows corresponds to two bagel disk boundaries with linking# $= 1$

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Main messages

- Physically knots-quivers correspondence is a duality between 3d $\mathcal{N}=2$ theories $T[L_K]$ and $T[Q_K]$
- Quiver elements can be intepreted in terms of $T[Q_K]$ data as well as holomorphic disks:

	Physics	Geometry
	$U(1)$ gauge group	Holomorphic disk
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- More details in the paper - coming soon!

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Last message

Thank you for your attention!