Counting instantons in N=1 theories of class S_k

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Quantum fields, knots and strings

Warsaw 25 September 2018

[1512.06079 Coman,EP,Taki,Yagi] [1703.00736 Mitev,EP] [1712.01288 Bourton,EP]

Motivation: N=2 exact results

- *Seiberg-Witten theory: effective theory in the IR*
- *Nekrasov: instanton partition function*
- *Pestun: observables in the UV (path integral on the sphere localizes)*

▷ String/M-/F-theory realizations

- *Gaiotto: 4D N=2* ***class S****: 6D (2,0) on Riemann surface* $C_{g,n}$
- *AGT: 4D partition functions = 2D CFT correlators*
- *4D SC Index = 2D correlation function of a TFT*

2D/4D relations

What can we do for N=1 theories?

- [x] *Superconformal Index* [Romelsberge 2005] [Kinney,Maldacena,Minwalla,Raju 2005]
- [x] *Intriligator and Seiberg: generalized SW technology*
- [x] *Witten: IIA/M-theory approach to curves*

Holomorphy fixes:
N=2 theories: prepotential (that's all in the IR)
N=1 theories: superpotential (there are also Kähler terms)

- [ ] *No Localization (No Nekrasov, no Pestun)*
- [ ] *An S^4 partition function plagued with scheme ambiguities.* [Gerchkovitz, Gomis, Komargodski 2014]
- [x] *Derivatives of the free energy scheme independent.* [Bobev, Elvang, Kol, Olson, Pufu 2014]

What can we do for N=1 theories?

- ☑ *Can construct **conformal** N=1 theories.* [Leigh,Strassler 1995]

- ☑ *AdS/CFT natural route to several examples.* [Kachru,Silverstein 1998] [Lawrence,Nekrasov,Vafa1998]

- ☑ *6D (1,0) on a Riemann Surface.* [Gaiotto,Razamat 2015] [Heckman,Vafa….]

Class S_k (S_Γ):

[Gaiotto,Razamat 2015]

2D/4D relation

- *Conformal*
- *Obtained by orbifolding N=2 (inheritance)*
- *Labeled by punctured Riemann Surface*
- *Index = 2D correlation function of a TFT*

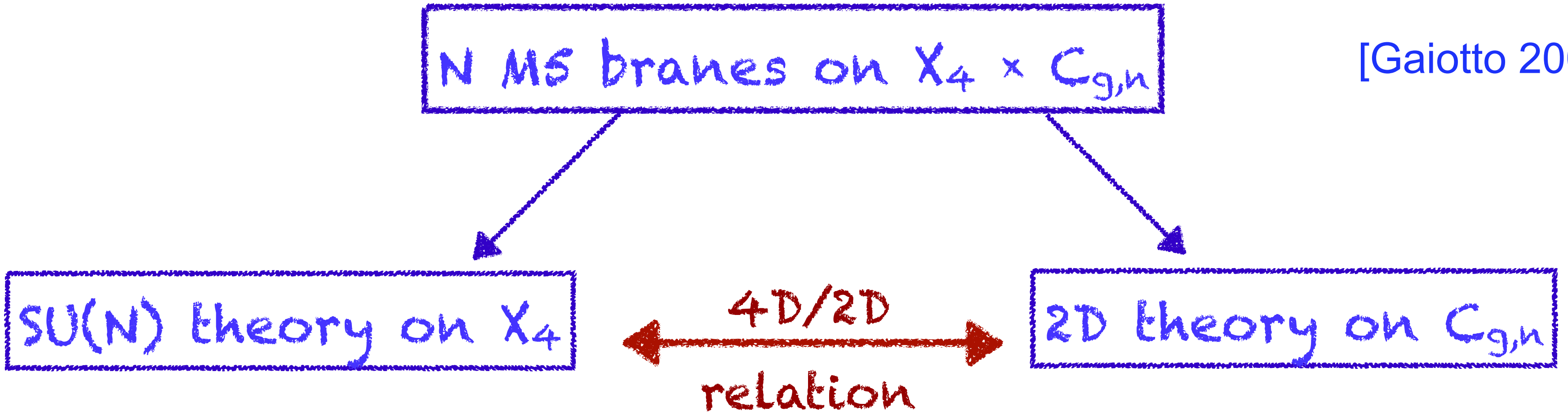
Plan

Is there AGT_k?
4D partition functions = 2D CFT correlators

- Introduce N=1 theories in class S_k
- Spectral curves for N=1 theories in class S_k
- From the curves: 2D symmetry algebra and representations
- Conformal Blocks ⟶ Instanton partition function
- Instanton partition function from Dp/D(p-4) branes on orbifold
- Free trinion partition functions on S^4 ⟶ 3pt functions

Class S_k

Class S and S_k



[Gaiotto 2009]

6D **(2,0)** SCFT on Riemann surface: 4D N=2 theories of **class S**

Transverse C^2/Z_k Orbifold the 6D (2,0) SCFT to 6D (1,0) SCFT

6D **(1,0)** SCFT on Riemann surface: 4D N=1 theories of **class S_k**

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}
N M5-branes	—	—	—	—	.	.	—	.	.	.	—
A_{k-1} orbifold	.	.	.	.	—	—	.	—	—	.	.

[Gaiotto,Razamat 2015]

Class S_k

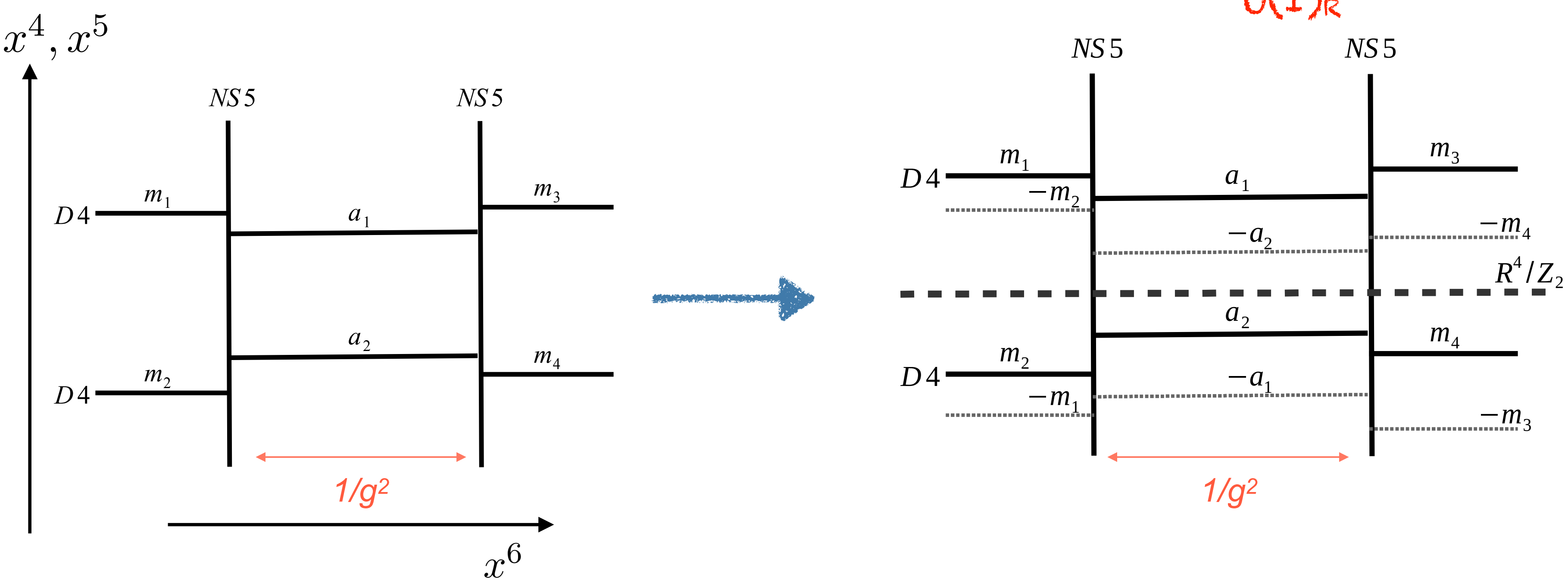
[Gaiotto,Razamat 2015]

Type IIA

$U(1)_r$ ~~$SU(2)_R$~~ of $\mathcal{N}=2$

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	(x^{10})
M NS5 branes	—	—	—	—	—	—	.	.	.	.	.
N D4-branes	—	—	—	—	.	.	—	.	.	.	—
A_{k-1} orbifold	.	.	.	.	—	—	.	—	—	.	.

$U(1)_R$



YM coupling: $1/g^2$

$$\epsilon = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\epsilon = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^6\epsilon = \Gamma^4\Gamma^5\Gamma^7\Gamma^8\epsilon$$

Class S_k

Type IIB

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
A_{M-1} orbifold	.	.	.	.	.	.	—	—	—	—
N D3-branes	—	—	—	—	.	.	.	.	.	.
A_{k-1} orbifold	.	.	.	.	—	—	.	—	—	.

- *N=1 orbifold daughter of N=4 SYM* $\Gamma = \mathbb{Z}_k \times \mathbb{Z}_M$

- *Useful for AdS/CFT (**orbifold inheritance**)* $AdS_5 \times \mathbb{S}^5 / \left(\mathbb{Z}_k \times \mathbb{Z}_M\right)$

  [Bershadsky, Kakushadze,Vafa 1998]

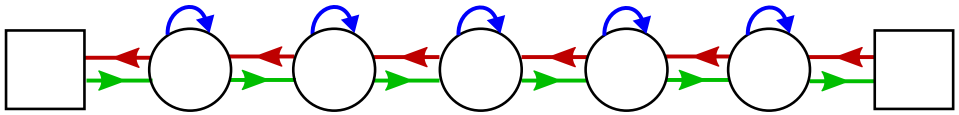
- *String theory technics to calculate instantons*

  [Dorey, Hollowood, Khoze, Mattis,…] [Lerda,…]

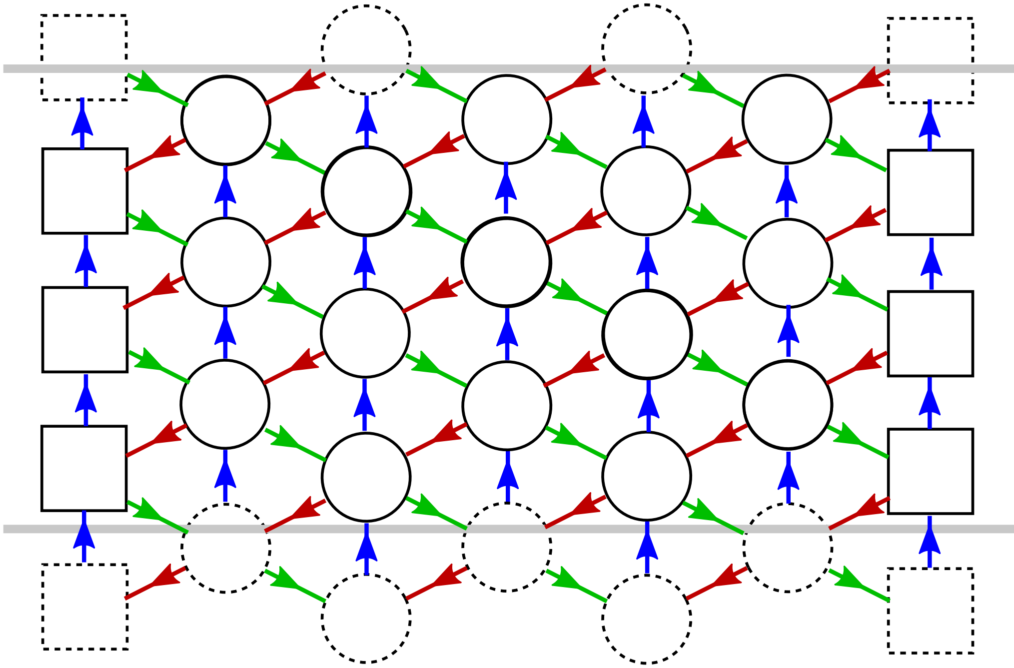
Class S_k

[Gaiotto,Razamat 2015]

4D field theory point of view



N=2 class S mother theory



N=1 class S_k daughter theory

	$U(1)_t$	$U(1)_{\alpha_c}$	$U(1)_{\beta_{i+1-c}}$	$U(1)_{\gamma_i}$
$V_{(i,c)}$	0	0	0	0
$\Phi_{(i,c)}$	-1	0	-1	$+1$
$Q_{(i,c-1)}$	$+1/2$	-1	$+1$	0
$\widetilde{Q}_{(i,c-1)}$	$+1/2$	$+1$	0	-1

Large global symmetry group

Class S_k

Begin with N=2 class S with SU(kN) gauge groups:

$$W_{\mathcal{S}} = \sum_{c=1}^{M-1} \left(Q_{(c-1)} \Phi_{(c)} \tilde{Q}_{(c-1)} - \tilde{Q}_{(c)} \Phi_{(c)} Q_{(c)} \right)$$

N=2 class S mother theory

<u>*Orbifold projection:*</u> [Douglas,Moore 1996]

$$Q_{(c)} = \begin{pmatrix} Q_{(1,c)} & & & \\ & Q_{(2,c)} & & \\ & & \ddots & \\ & & & Q_{(k,c)} \end{pmatrix}$$

$$\underset{kN\times kN}{\Phi_{(c)}} = \begin{pmatrix} & \Phi_{(1,c)} & & & \\ & & \Phi_{(2,c)} & & \\ & & & \ddots & \\ & & & & \Phi_{(k-1,c)} \\ \underset{N\times N}{\Phi_{(k,c)}} & & & & \end{pmatrix}$$

$$\tilde{Q}_{(c)} = \begin{pmatrix} & & & \tilde{Q}_{(k,c)} \\ \tilde{Q}_{(1,c)} & & & \\ & \ddots & & \\ & & \tilde{Q}_{(k-1,c)} & \end{pmatrix}$$

$$W_{\mathcal{S}_k} = \sum_{i=1}^{k} \sum_{c=1}^{M-1} \left(Q_{(i,c-1)} \Phi_{(i,c)} \tilde{Q}_{(i,c-1)} - \tilde{Q}_{(i,c)} \Phi_{(i,c)} Q_{(i+1,c)} \right)$$

N=1 class S_k daughter theory

Coulomb and Higgs branch

N=2 class S mother theory

* **Coulomb Branch:** $\langle Q\rangle = 0$ *with* $\langle\phi\rangle = \mathrm{diag}\,(a_1,\ldots,a_N)$ *and* $u_\ell = \langle \mathrm{tr}\phi^\ell\rangle$

  $E = r$

* **Higgs Branch:** $\langle\phi\rangle = a = 0$ *with operators* $E = 2R$ *e.g.*

  $m_i = 0$

  $\mu_{\mathcal{I}\mathcal{J}} = \langle \mathrm{tr}\,(Q_{\{\mathcal{I}}\bar{Q}_{\mathcal{J}\}})\rangle$

N=1 class S_k daughter theory

* **Coulomb Branch:** $\langle Q\rangle = 0$ *parameterised by* $u_{\ell k} = \langle \mathrm{tr}\left(\Phi_{(1)}\cdots\Phi_{(k)}\right)^\ell\rangle$

  $\langle U^{-1}\Phi U\rangle = \mathrm{diag}\,(a_1,a_2,\cdots,a_N)\otimes\mathrm{diag}\left(1,e^{\frac{2\pi i}{k}},e^{\frac{4\pi i}{k}}\cdots e^{\frac{2\pi i(k-1)}{k}}\right)$

  $E = r$

* **Higgs Branch:** *similar w/ mother theory, operators charged under new beta and gamma symmetries.*

CB and HB ***do not mix*** *(no relations): charged under different charges!*

[Bourton,Pini,EP to appear]

Curves

[1512.06079 Coman,EP,Taki,Yagi]

Generic N=1 Curves

* *The spectral curve computes the effective YM coupling constants.*

[Intriligator,Seiberg]

* *M-theory on* $\mathbb{R}^{3,1} \times CY_3 \times \mathbb{R}^1$ [Bah, Beem, Bobev, Wecht]

CY_3 locally two holomorphic line bundles on the curve $C_{g,n}$

* *N=1 spectral curve is an overdetermined algebraic system of eqns.*

[Bonelli,Giacomelli,Maruyoshi,Tanzini]

[Xie…]

* *For class S_k on the Coulomb Branch (* $\langle Q \rangle = 0$ *) only one equation*

exactly like for N=2 theories.

[Coman,EP,Taki,Yagi]

S_k curves

[Coman,EP,Taki,Yagi]

$v = x^4 + ix^5$

$w = x^7 + ix^8$

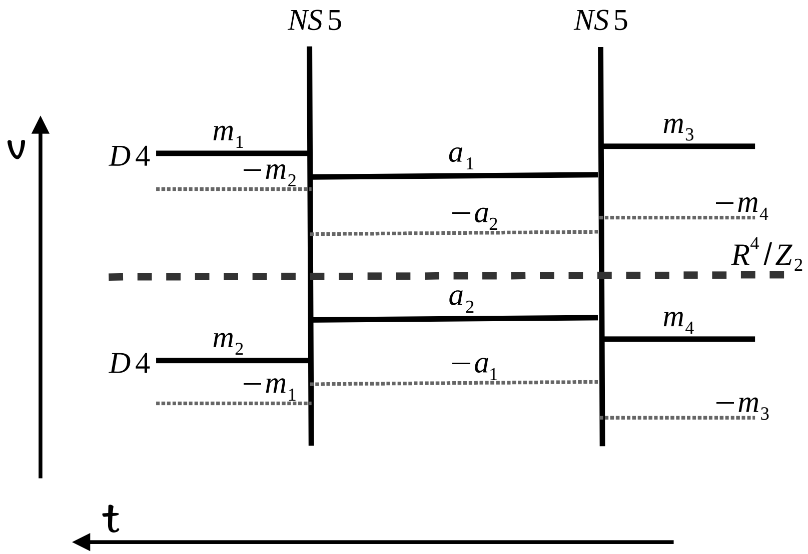
$t = e^{-\frac{x^6 + ix^{10}}{R_{10}}}$

$U(1)_r$ ~~$SU(2)_R$~~ of N=2

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	(x^{10})
M NS5 branes	−	−	−	−	−	−	.	.	.	.	.
N D4-branes	−	−	−	−	.	.	−	.	.	.	−
A_{k-1} orbifold	.	.	.	.	−	−	.	−	−	.	.

$U(1)_R$

$$(v\,,\,w) \sim \left(e^{\frac{2\pi i}{k}} v\,,\, e^{-\frac{2\pi i}{k}} w\right)$$



Take all the positions of all the branes on the w plane to be at w=0

[Lykken,Poppitz,Trivedi 97]

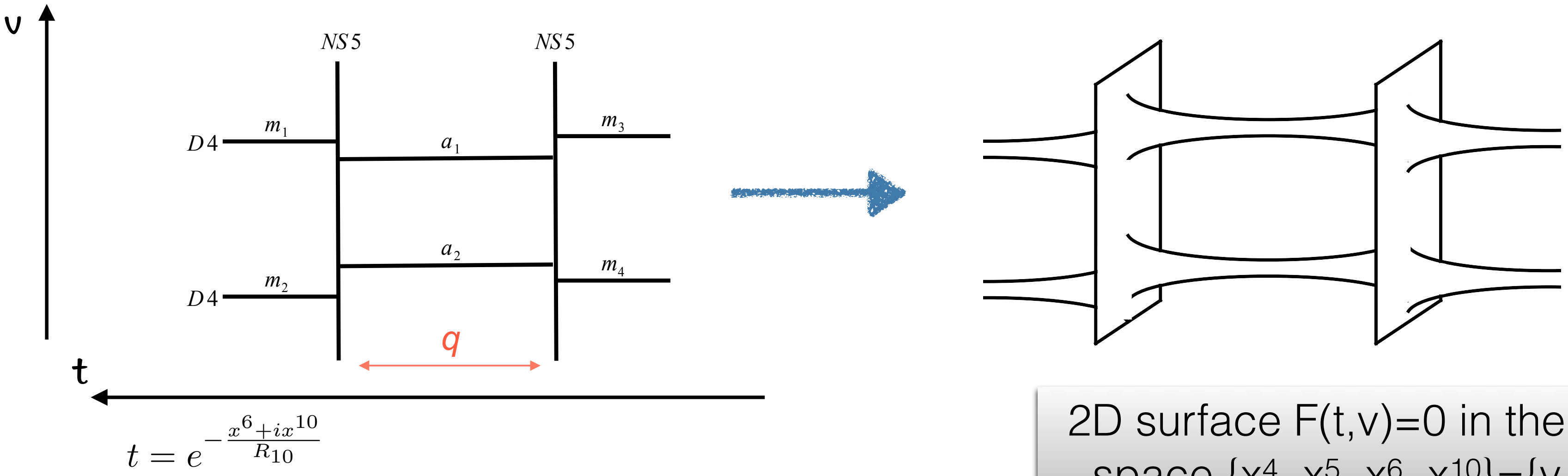
$$\langle Q \rangle = 0$$

Zero vevs for Higgs branch operators!

Curves from M-theory

[Witten 1997]

$v = x^4 + ix^5$



2D surface F(t,v)=0 in the 4D space {x^4, x^5, x^6, x^{10}}={v,t}.

The NS5/D4 is the classical configuration.

Take in account tension of the branes: include quantum effects.

M-theory: a single M5 brane with non trivial topology

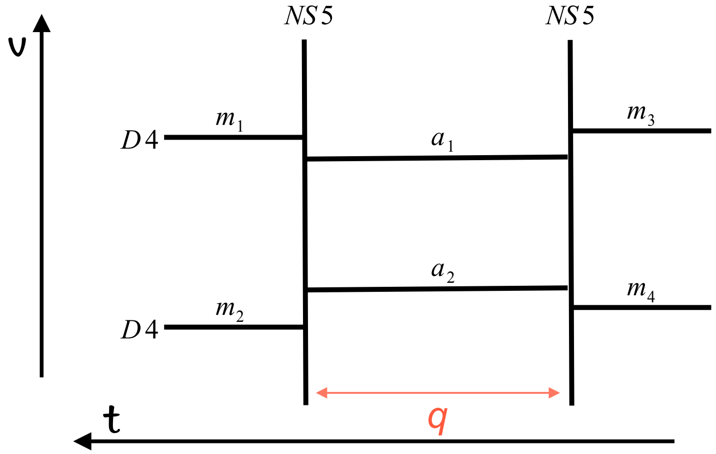
$$(t-1)(t-q)v^2 - P_1(t)v + P_2(t) = 0$$

$$(v-m_1)(v-m_2)t^2 + \left(-(1+q)v^2 + qMv + u\right)t + q(v-m_3)(v-m_4) = 0$$

coupling constant $q = e^{2\pi i \tau_{UV}}$ $u = \mathrm{tr}\phi^2$ $M = m_1+m_2+m_3+m_4$

Class S Curve

SU(2) with 4 flavors



$$(t-1)(t-q)v^2 - P_1(t)v + P_2(t) = 0$$

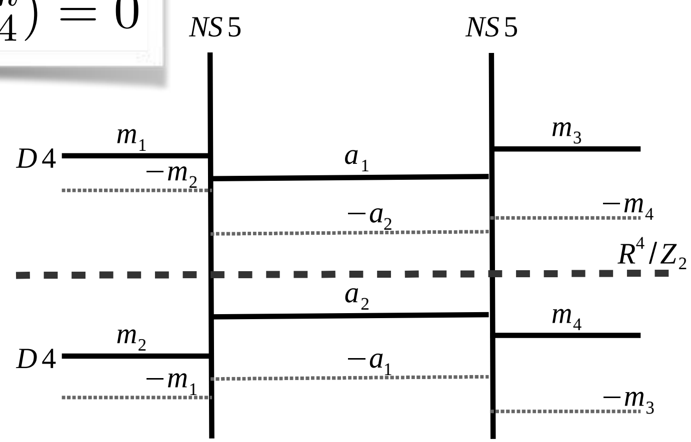
$$(v-m_1)(v-m_2)t^2 + \left(-(1+q)v^2 + qMv + u\right)t + q(v-m_3)(v-m_4) = 0$$

$q = e^{2\pi i \tau_{UV}}$ $\quad u = \mathrm{tr}\,\phi^2$ $\quad M = m_1 + m_2 + m_3 + m_4$

Class S$_k$ Curve

$$v \sim e^{\frac{2\pi i}{k}} v$$

$$(v^k - m_1^k)(v^k - m_2^k)t^2 + P(v)t + q(v^k - m_3^k)(v^k - m_4^k) = 0$$



$$P(v) = -(1+q)v^{2k} + u_k v^k + u_{2k}$$

vevs of gauge invariant operators: parameterize the Coulomb branch

$$\langle \mathrm{tr}\left(\Phi_{(1)} \cdots \Phi_{(k)}\right) \rangle \sim u_k \qquad \langle \mathrm{tr}\left(\Phi_{(1)} \cdots \Phi_{(k)}\right)^2 \rangle \sim u_{2k}$$

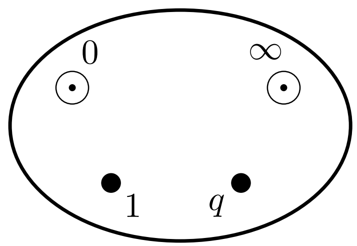
S_k curves

SW or IR curve Σ of g=kN-1 → Gaiotto or UV curve $C_{0,n}$ a sphere with n punctures

$$x^{kN} = -\sum_{\ell=1}^{N} \phi^{(4)}_{k\ell}(t)\, x^{k(N-\ell)}$$

$$\mathfrak{c}^{(s,k)} = \sum_{i_1<\cdots<i_s=1}^{N} m_{i_1}^k \cdots m_{i_s}^k$$

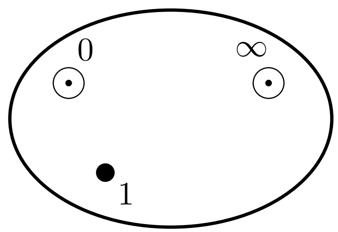
$$\phi^{(4)}_{k\ell}(t) = \frac{(-1)^\ell\, \mathfrak{c}_L^{(\ell,k)}\, t^2 + u_{k\ell} t + (-1)^\ell\, \mathfrak{c}_R^{(\ell,k)}\, q}{t^{k\ell}(t-1)(t-q)}$$



$\mathcal{N}=1$ SU(N) SCQCD$_k$

- ⊙ **Full/maximal puncture**: k images of N mass parameters
- • **Simple puncture**: No further parameters

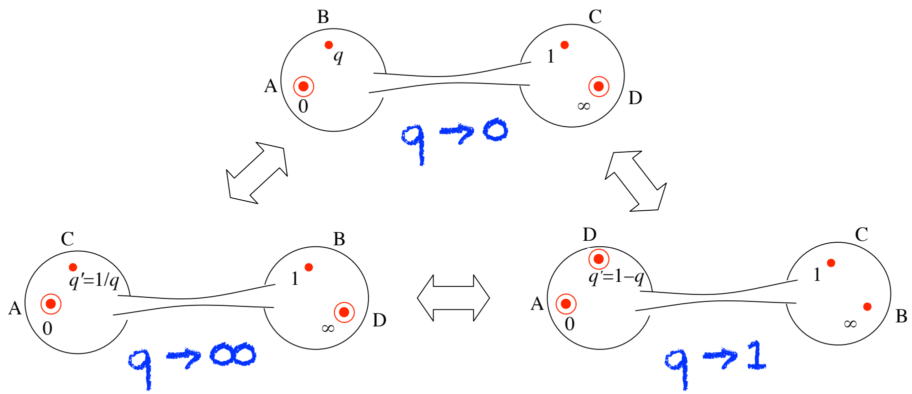
$$\phi^{(3)}_{k\ell}(t) = (-1)^\ell \frac{\mathfrak{c}_L^{(\ell,k)}\, t - \mathfrak{c}_R^{(\ell,k)}}{t^{k\ell}(t-1)}$$

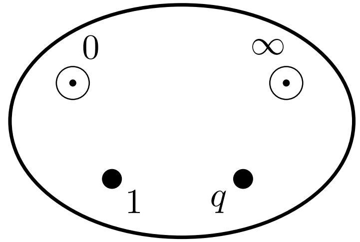


Free Trinion $\mathcal{C}^{(k)}_{0,3}$

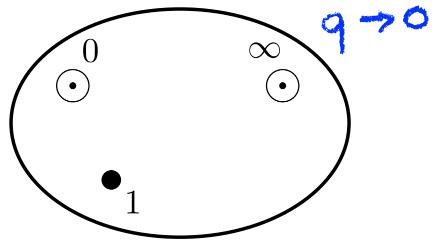
Curve decomposition and S-duality

* *Different S-duality frames (as for N=2 class S)*



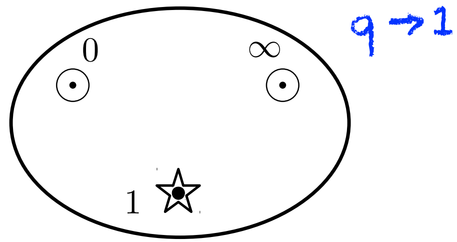


$\mathcal{N}=1$ SU(N) SCQCD$_k$



Free Trinion $\mathcal{C}_{0,3}^{(k)}$

* *Weak coupling: free trinions*



* *Strong coupling limit: new type of punctures!*

S-duality = 2D crossing equation

[Coman,EP,Taki,Yagi]

Instantons from the 2D Blocks

$$\mathcal{Z}_{\text{inst}} = \mathcal{B}_{\mathbf{w}}(\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4 | q)$$

[1703.00736 Mitev,EP]

The AGT-W correspondence

[Alday,Gaiotto,Tachikawa] [Wyllard]

A relation between:

- 4D N=2 theories of class S with SU(2)/SU(N) factors
- 2D Liouville/Toda CFT

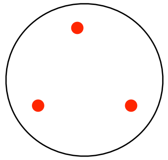
$$\mathcal{Z}_{\mathbb{S}^4}\left[\mathcal{T}_{g,n}\right] = \int da\, \mathcal{Z}_{pert}\left|\mathcal{Z}_{inst}\right|^2 = \int d\alpha\, C \ldots C \left|\mathcal{B}_{\alpha}^{\alpha_i}\right|^2 = \langle \prod_{i=1}^{n} V_{\alpha_i} \rangle_{\mathcal{C}_{g,n}}$$

4D gauge theory	2D CFT
instanton partition function	conformal block
perturbative part	3-point function
coupling constants	cross ratios
masses	external momenta
Coulomb moduli	internal momenta
generalized S-duality	crossing symmetry
Omega background	Coupling constant/central charge

$$b^2 = \frac{\epsilon_1}{\epsilon_2}$$

The AGT relation from the curve

Example:
N=2 SU(2) Free trinion



Close to the punctures:

$$\phi_2^{(3)}(z) \sim \frac{m_j^2}{(z-z_j)^2}$$

Recall 2D Ward Identities:

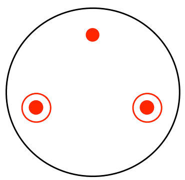
$$\langle T(z)\mathsf{V}_1(z_1)\mathsf{V}_2(z_2)\mathsf{V}_3(z_3)\rangle = \sum_{j=1}^{3}\left[\frac{h_j}{(z-z_j)^2} + \frac{\partial_j}{z-z_j}\right]\langle \mathsf{V}_1(z_1)\mathsf{V}_2(z_2)\mathsf{V}_3(z_3)\rangle$$

$$h_i = -m_i^2$$

$$\phi_2^{(3)}(z) = \frac{\langle\, T(z)\mathsf{V}_1(z_1)\mathsf{V}_2(z_2)\mathsf{V}_3(z_3)\,\rangle}{\langle\, \mathsf{V}_1(z_1)\mathsf{V}_2(z_2)\mathsf{V}_3(z_3)\,\rangle}$$

Free trinions curves are equivalent to Ward identities!

N=2 SU(N)
Free trinion



$$\phi_\ell^{(3)}(z) = \frac{\langle\, W_\ell(z)\mathsf{V}_\odot(z_1)\mathsf{V}_\bullet(z_2)\mathsf{V}_\odot(z_3)\,\rangle}{\langle\, \mathsf{V}_\odot(z_1)\mathsf{V}_\bullet(z_2)\mathsf{V}_\odot(z_3)\,\rangle}$$

From the curves to the 2D CFT

$$\lim_{\epsilon_{1,2}\to 0} \langle\langle\, J_\ell(t)\,\rangle\rangle_n = \phi_\ell^{(n)}(t)$$

$$\langle\langle\, J(t)\,\rangle\rangle_n \stackrel{\text{def}}{=} \frac{n\text{-point W-block with insertion of } J(t)}{n\text{-point W-block}}$$

* *The symmetry algebra that underlies the 2D CFT = W_{kN} algebra*

* *The reps are **standard** reps of the W_{kN} algebra*

* *Obtain them from the N=2 SU(kN) after replacing:*

$$m_{j+Ns}^{\mathrm{SU}(Nk)} \longmapsto m_j\, \mathrm{e}^{\frac{2\pi i}{k}s} \qquad a_{j+Ns}^{\mathrm{SU}(Nk)} \longmapsto a_j\, \mathrm{e}^{\frac{2\pi i}{k}s}$$

[1703.00736 Mitev,EP]

2D Conformal Blocks = Instanton P.F.

* *We have the reps of the W_{kN} algebra for $\varepsilon_{1,2} = 0$ (from the curve)*

* *Demand: the structure of the multiplet (null states) not change $\varepsilon_{1,2} \neq 0$*

* *The blocks for $\varepsilon_{1,2} \neq 0$: proposal for the instanton partition functions:*

$$\mathcal{Z}_{\text{inst}} = \mathcal{B}_{\mathbf{w}}(\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4 | q)$$

* *If w and c turn on $Q \neq 0$ as in Liouville/Toda,*

  then we obtain them from the N=2 SU(kN) after replacing:

$$m_{j+Ns}^{\mathrm{SU}(Nk)} \longmapsto m_j\, \mathrm{e}^{\frac{2\pi i}{k} s} \qquad a_{j+Ns}^{\mathrm{SU}(Nk)} \longmapsto a_j\, \mathrm{e}^{\frac{2\pi i}{k} s}$$

[1703.00736 Mitev,EP]

Instantons from D(p-4) branes

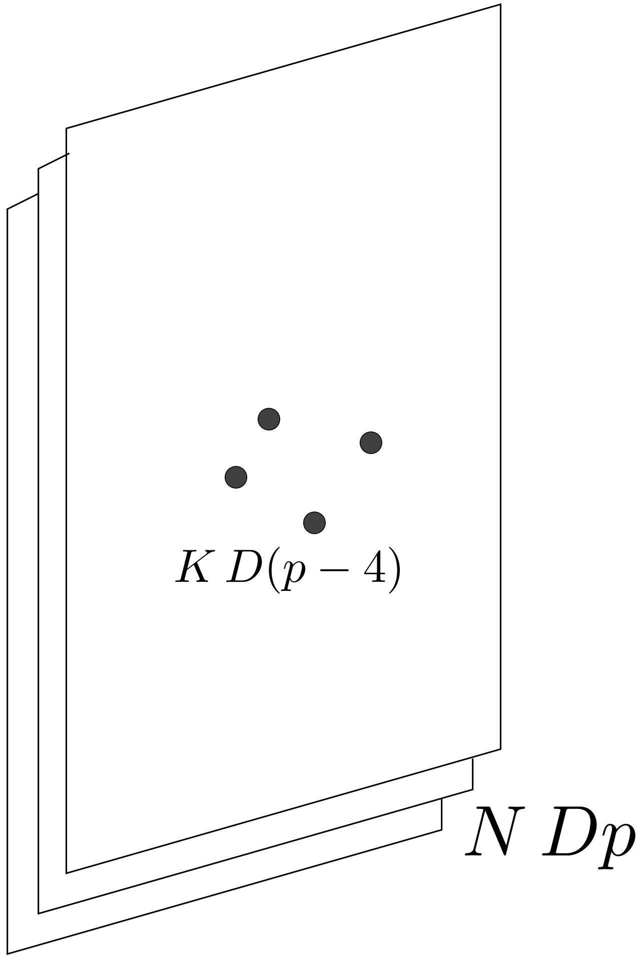
[1712.01288 Bourton, EP]

Instantons from D branes

$$\mathrm{Tr} \int_{Dp} d^{p+1}x \;\; C_{p-3} \wedge F \wedge F$$

Instanton on Dp brane = D(p-4) brane

$$\mu^{\mathbf{C}} := [B_1, B_2] + IJ = 0 \; ,$$
$$\mu^{\mathbf{R}} := [B_1, B_1^\dagger] + [B_2, B_2^\dagger] + II^\dagger - J^\dagger J = 0$$



The Dp-D(p-4) strings give the NxK I, J†
and the D(p-4)-D(p-4) the KxK B₁, B₂
auxiliary matrices of the ADHM
construction

$$\mathcal{M}^{Dp}_{K\text{-inst}} \simeq \mathcal{M}^{D(p-4)}_{\text{Higgs}}$$

$$= \left\{ B_1, B_2, I, J \,|\, \mu^{\mathbf{C}} = 0, \mu^{\mathbf{R}} = 0 \right\} / U(K)$$

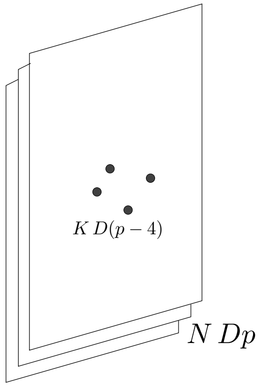
$$= \left\{ X^{\perp}_{Dp} = 0, \mathcal{V}_{D(p-4)} = 0 \right\} / U(K)$$

[Witten 1995, Douglas 1995, Dorey 1999, ...]

Instanton Partition Function from 2D SCI

$$\mathcal{Z}_{K\text{-inst}}(a,m,\epsilon_1,\epsilon_2) \;=\; \mathrm{Tr}_{\mathcal{M}_{K\text{-inst}}}\underline{(-1)^F e^{\epsilon_1 J_L} e^{\epsilon_2 J_R} e^{\xi R} e^{a J_G} e^{m J_F}}$$

Nekrasov's Instantons: deformed Witten index



$$\mathcal{M}^{Dp}_{K\text{-inst}} \simeq \mathcal{M}^{D(p-4)}_{\text{Higgs}}$$

For p=5 and when the D1s wrapping a T^2:

Instanton's index = 2D SCI = flavoured elliptic genius

of the 2D gauge theory living on the D1s

2D R-symmetry

$J_{69}-J_{78}$

J_{01} , J_{23}

$$\mathcal{Z}^{\text{D5}}_{K\text{-inst}}(a,m,\epsilon_1,\epsilon_2) \;=\; \mathcal{Z}^{K\ \text{D1}}_{\text{Higgs}}(a,m,\epsilon_1,\epsilon_2) = \mathcal{I}_{2D} \;=\; \mathrm{Tr}_{\mathcal{M}^{K\ D1}_{\text{Higgs}}}(-1)^F e^{\epsilon_1 J_L} e^{\epsilon_2 J_R} e^{\xi R} e^{a J_G} e^{m J_F}$$

$J_{69}+J_{78}$ 2D Flavor

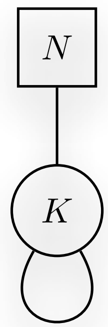
2D SCI is very well studied, very easy to compute!

[Gadde, Gukov, Putrov]
[Benini,Eager,Hori,Tachikawa]

Mass deformed N=4 SYM

First we practice without the orbifolds:

- *Obtain the $\mathcal{Z}_{\text{inst}}$ for mass deformed N=4 SYM (N=2*)*
  from a ***2D SCI computation:*** 2D theory with (4,4) susy



$$D3/D(-1) \xleftrightarrow{\text{T-duality}} D5/D1$$

- *The 2D SCI of the gauge theory on the D1 branes*
  = instantons of the 6D (2,0) on $\mathbb{R}^4 \times T^2$ *= M-strings.*

- *KK reducing: instantons of mass deformed 5D N=2 MSYM on* $\mathbb{S}^1$
  and further KK reduce to mass deformed N=4 SYM in 4D.

Instantons with an orbifold

* *"**Orbifold**" the 2D SCI of mass deformed N=4 SYM with one, say the Z_M orbifold: we get M-strings on a transverse orbifold*

  = instantons of an $SU(N)^M$ quiver when reduce down to 5D/4D

  2D theory with (0,4) susy

* *Further "**Orbifold**" the 2D SCI with the Z_k orbifold we get something new that should correspond to instantons of class Sk,*

  the rational, trigonometric and elliptic uplift.

  2D theory with (0,2) susy

S_k Instantons

$$\mathcal{Z}^{\rm D5}_{K\text{-inst}}(a,m,\epsilon_1,\epsilon_2) = \mathcal{Z}^{K\ \rm D1}_{\rm Higgs}(a,m,\epsilon_1,\epsilon_2) = \mathcal{I}_{2D} = \mathrm{Tr}_{\mathcal{M}^{K\ D1}_{\rm Higgs}}(-1)^F e^{\epsilon_1 J_L} e^{\epsilon_2 J_R} e^{\xi R} e^{aJ_G} e^{mJ_F}$$

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
N D5	–	–	–	–	–	–	·	·	·	·
$\mathbb{Z}_M$	·	·	·	·	·	·	×	×	×	×
$\mathbb{Z}_k$	·	·	·	·	×	×	·	×	×	·
K D1	·	·	·	·	–	–	·	·	·	·

$\mathbb{R}^4 \times T^2$ $\qquad$ $\mathbb{R}^4 \times \mathbb{S}^1$ $\qquad$ $\mathbb{R}^4$

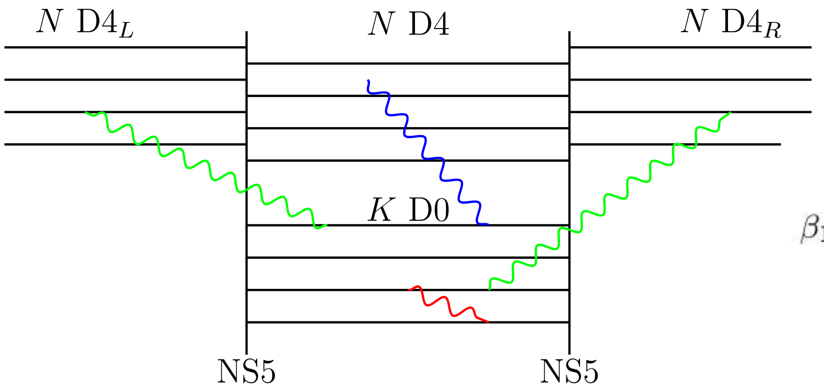
$$\mathcal{I}_{D1} = \mathcal{Z}^{(2,0)}_{\rm inst} \xrightarrow{\rm KK} \mathcal{Z}^{\mathcal{N}=2}_{\rm inst} \xrightarrow{\rm KK} \mathcal{Z}^{\mathcal{N}=4}_{\rm inst}$$

$$\mathcal{I}^{\mathbb{Z}_M}_{D1} = \mathcal{Z}^{(1,0)}_{\rm inst} \xrightarrow{\rm KK} \mathcal{Z}^{\mathcal{N}=1}_{\rm inst} \xrightarrow{\rm KK} \mathcal{Z}^{\mathcal{N}=2}_{\rm inst}$$

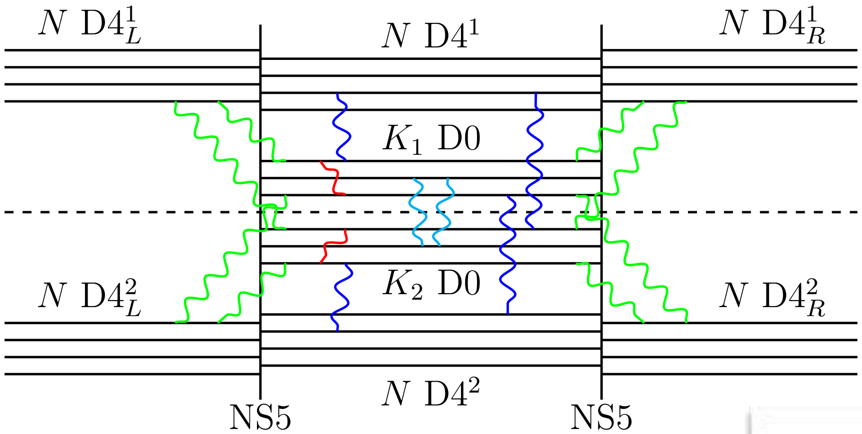
$$\mathcal{I}^{\mathbb{Z}_M\times\mathbb{Z}_k}_{D1} = \mathcal{Z}^{(1,0)\rm w/defect}_{\rm inst} \xrightarrow{\rm KK} \mathcal{Z}^{\mathcal{N}=1\rm w/defect}_{\rm inst} \xrightarrow{\rm KK} \boxed{\mathcal{Z}^{\mathcal{N}=1}_{\rm inst}}$$

✱ *The same answer as in* [1703.00736 Mitev,EP] *from conformal blocks!*

Instantons from D0 branes



$$\lim_{\beta_1,\beta_2\to 0} I_{SCI} = \frac{1}{K!}\int \prod_{I=1}^{K} du_I \prod_{I,J=1}^{K} \frac{u'_{IJ}\,(u_{IJ}-2\epsilon_+)}{(u_{IJ}+\epsilon_1)\,(u_{IJ}+\epsilon_2)} \times \prod_{I=1}^{K}\prod_{A=1}^{N} \frac{(u_I-m_{L,A})\,(u_I-m_{R,A})}{(u_I-a_A-\epsilon_+)\,(u_I-a_A+\epsilon_+)}$$



$$\lim_{\beta_1,\beta_2\to 0} I^{\mathrm{orb}}_{SCI} \propto \prod_{i=1}^{k} \int \prod_{I=1}^{K_i} du_{i,I} \prod_{I=1}^{K_i} \frac{u'_{ii,IJ}\prod_{j\neq i}\prod_{J=1}^{K_j}(u_{ij,IJ}-2\epsilon_+)}{\prod_{j=1}^{k}\prod_{J=1}^{K_j}(u_{ij,IJ}+\epsilon_1)\,(u_{ij,IJ}+\epsilon_2)} \times \prod_{j=1}^{k}\prod_{I=1}^{K_i}\prod_{A=1}^{N} \frac{(u_{i,I}-\widetilde{m}_{L,j,A})\,(u_{i,I}-\widetilde{m}_{R,j,A})}{(u_{i,I}-\widetilde{a}_{j,A}-\epsilon_+)\,(u_{i,I}-\widetilde{a}_{j,A}+\epsilon_+)}$$

The Perturbative piece

Free trinion P.F. = 2D CFT 3pt functions

[in progress Carstensen,EP,Mitev]

$$\mathcal{Z}^{S^4}_{\text{free trinion}} = \langle \, \mathsf{V}_{\odot}(\infty)\mathsf{V}_{\bullet}(1)\mathsf{V}_{\odot}(0) \, \rangle$$

- *The free trinion theory on S^4 : we can do the determinant!*
- *We know the conformal blocks: can write crossing equations*
- *Is the free trinion P.F. a solution of the crossing equations ??*
- *3pt functions (dynamics) + Blocks = AGT_k*

The Perturbative part

- *No Localization: No Pestun for N=1 susy!*

- *The free trinion theory on S^4 : we can do the determinant!*
  [in progress Carstensen,EP,Mitev]

- *Guess via analytic continuation from 3D* [Gorantis,Minahan,Naseer]
  The orbifolded hyper part agrees with our determinant calculation!

- *Deconstruction of (1,1) Little Strings* [Hayling,Panerai,Papageorgakis]
  vector and chiral contributions precisely give the perturbative Little string!

We can write a reasonable guess for the perturbative part!

Summary

- ☑ *We constructed spectral* ***curves*** *for N=1 theories in class* S_k
- ☑ *The curves: 2D symmetry algebra* (W_{kN}) *and representations*
- ☑ *Conformal Blocks* ⟶ *Instanton partition function*
- ☑ *Instanton partition function from Dp/D(p-4) on orbifold*
- ☐ *Free trinion partition functions on* S^4 = ***3pt functions***

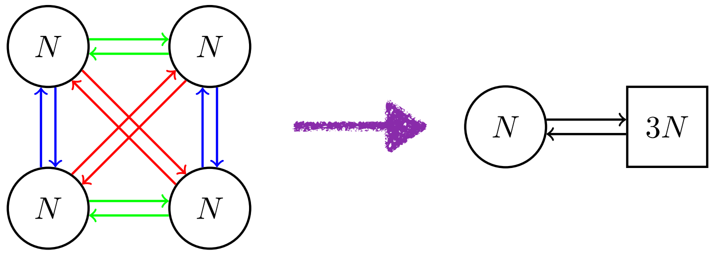
[in progress Carstensen,EP,Mitev]

Future

- *Compute one, two instantons with standard QFT techniques*

- *Go away from the orbifold point*

  [to appear Bourton, EP]



- *Other N=1 theories*

- *The perturbative part? N=1 partition function on S^4!*

- *Get the AGT_k from (1,0) 6D à la Cordova and Jafferis*

Thank you!